Sorting on GPUs

Some not-so-good sorting approaches

Bitonic sort

QuickSort

Concurrent kernels and recursion

Adapt algorithms to parallel execution

Many sorting algorithms are highly sequential

Suitable for parallel implementation?

- Data driven execution
- Data independent execution

Data driven execution

Computing pattern depends on data

Usually harder to parallellize!

Example: QuickSort.

Data independent execution

Known computing pattern

Easier to parallellize - always the same plan

Example: Bitonic sort

Bubble sort

Loop through data, compare neighbors

Extremely sequential

Inefficient

Parallel version: Bubble sort with odd-even transposition method

Compare all items pairwise

Two phases, ”odd phase” and ”even phase” (shifted one step)

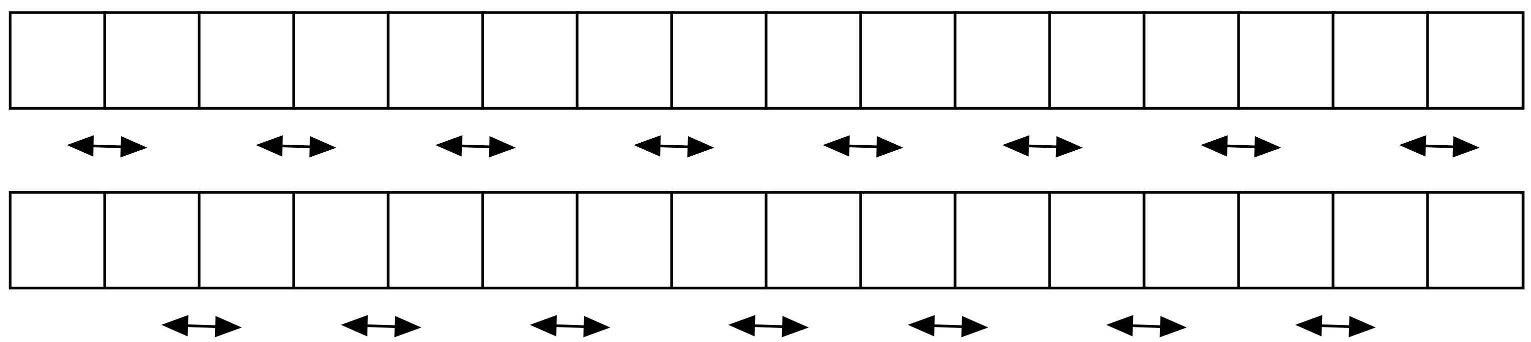
Bubble sort, parallel version

Bubble sort with odd-even transposition method

Compare all items pairwise

Two phases, ”odd phase” and ”even phase” (shifted one step)

Fully sorted after n phases



Even phase

Odd phase

$O(n^2)$

Suitable for GPU?

Not as bad as it seems at first look:

- Data independent
- Excellent locality
- Appears to have possibilities to use shared memory but with some costly transfers at edges between blocks.
- But certainly not optimal at very large sizes

Perfect for sorting many small sets but not one large!

”Better” algorithms don’t necessary beat this all that easily!

Rank sort

Count number of items that are smaller

Values must be unique!

Easy to parallelize:

- One thread per item
- Loop through entire data
- Store in index decided from count of number of smaller items.

Suitable for GPU?

Again, not as bad as it seems at first look:

- Data independent
- Excellent locality - especially good for broadcasting (e.g. constant memory). Also suitable for shared memory.
- Again, $O(n^2)$: Will grow at very large sizes

Two bad ones that are not quite as bad as they seem.

N parallel iterations may beat NlogN sequential ones!

Just as exercise

Rank sort optimization

Everybody want to know what rank they have.

They all need to compare to everything.

For each block of N threads

Split memory in chunks of N

Read chunk shared, one per thread

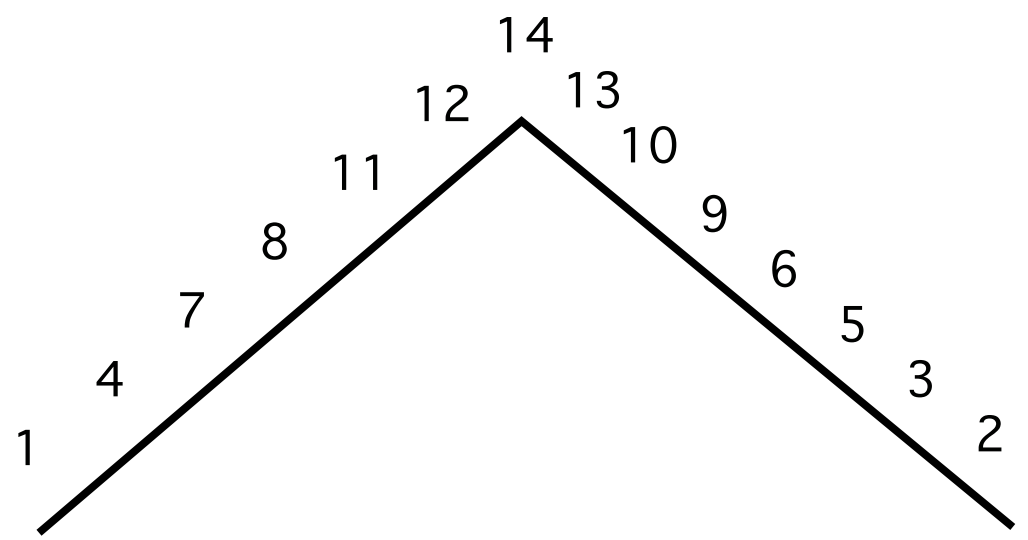
Synchronize

Read through chunk in shared

Writing result is conflict free

Bitonic merge sort

Bitonic set: Two monotonic parts in different direction.



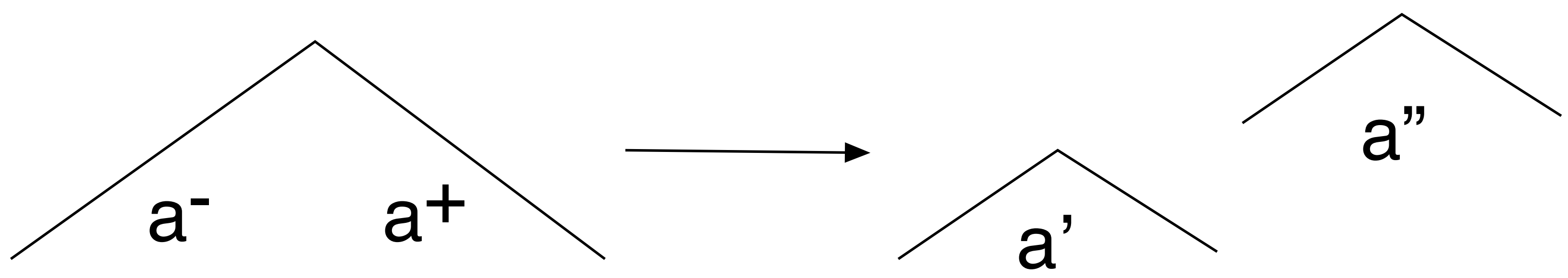
Bitonic merge sort

(According to Batcher:) Let a be a bitonic set with a maximum at k, consisting of two monotonic parts, one increasing, a^- (from item 1 to k) and one decreasing, a^+ (k+1 to n)

Then two new sets can be constructed as

$$a' = \min(a_1, a_{k+1}), \min(a_2, a_{k+2})\ldots$$
$$a'' = \max(a_1, a_{k+1}), \max(a_2, a_{k+2})\ldots$$

These two sets are also bitonic and max(a’) ≤ min(a”)!

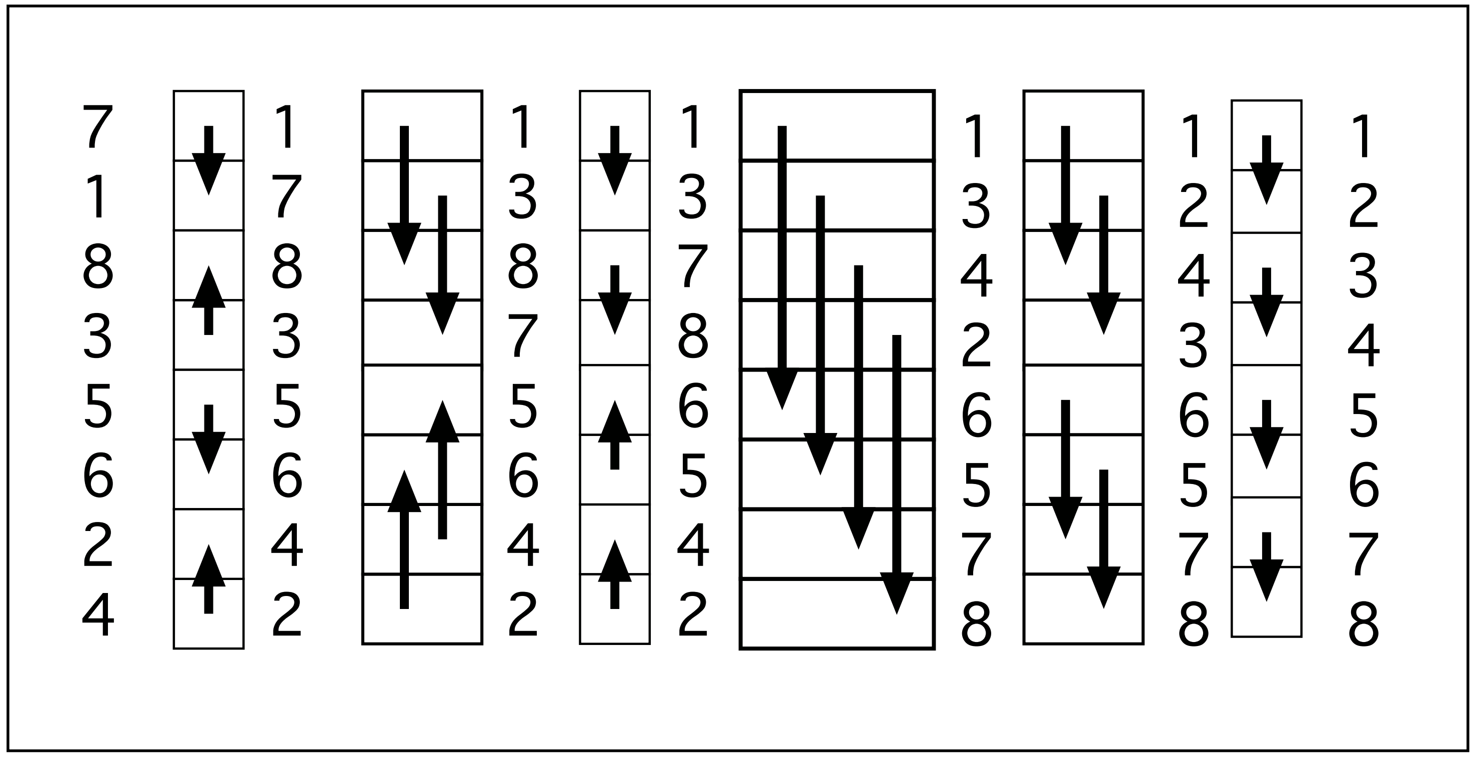


Bitonic sort by divide-and-conquer

Bitonic sort works on a bitonic sequence: partially sorted

The parts must be sorted. Sort them by bitonic sort!

Bitonic sort example



Bitonic sort of smaller parts

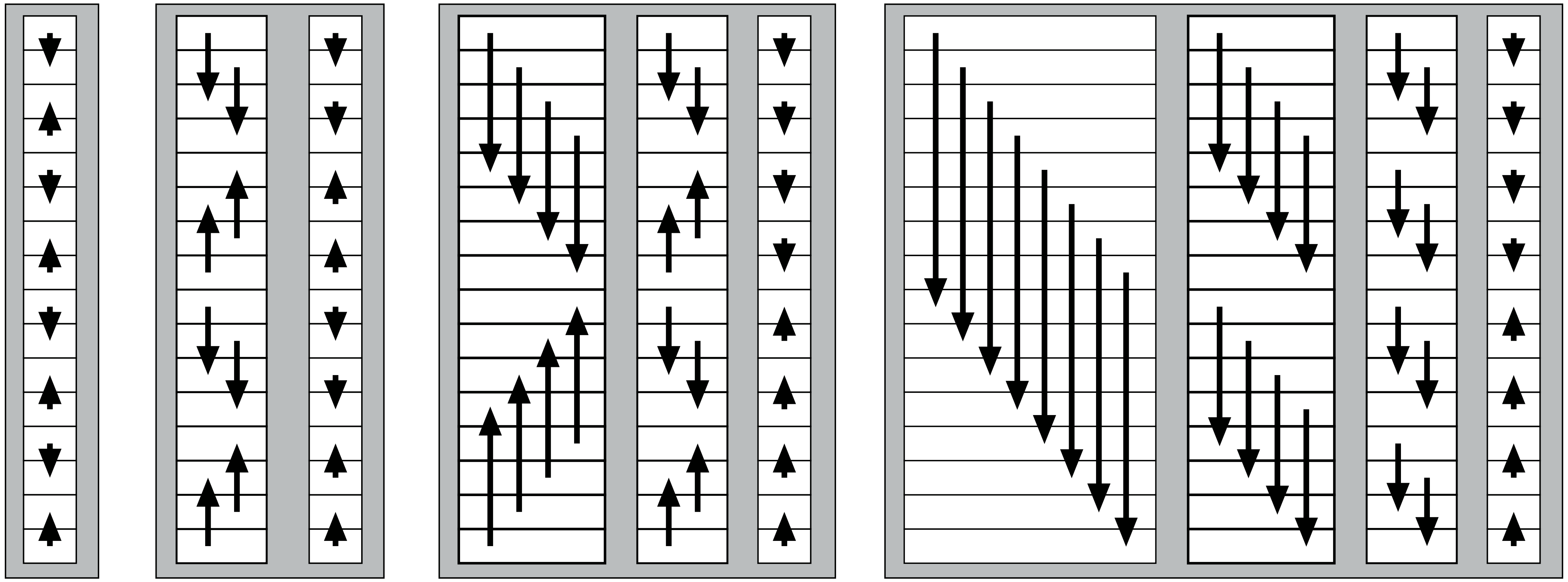
Reverse parts (bitonic merge)

Bitonic sort of main part

Reverse parts (bitonic merge)

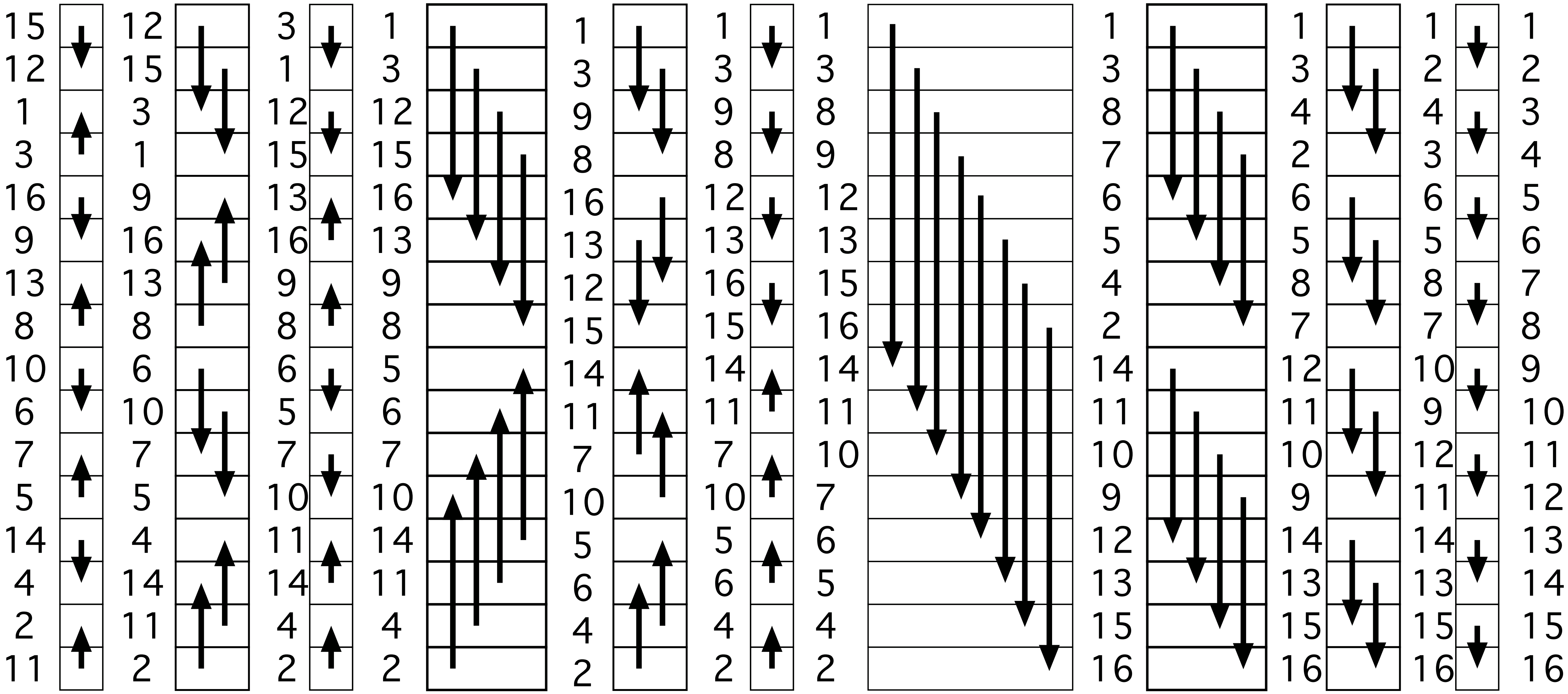
Bigger example

The problem scales nicely, uniformly



More stages gives longer stages

(Image inspired by one from Wikipedia)


15 12 1 3 16 9 13 8 10 6 7 5 14 4 2 11
12 15 3 1 9 16 13 8 6 10 7 5 4 14 11 2
3 1 12 15 13 16 9 8 6 5 7 10 11 14 4 2
1 3 12 15 16 13 9 8 5 6 7 10 14 11 4 2
1 3 9 8 16 13 12 15 14 11 7 10 5 6 4 2
1 3 9 8 12 13 16 15 14 11 7 10 5 6 4 2
1 3 8 9 12 13 15 16 14 11 10 7 6 5 4 2
1 3 8 7 6 5 4 2 14 11 10 9 12 13 15 16
1 3 4 2 6 5 8 7 12 11 10 9 14 13 15 16
1 2 4 3 6 5 8 7 10 9 12 11 14 13 15 16
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

Get those steps right

Step length

Step direction

Comparison direction

Calculated from stage number and stage length

Code examples

Sequential:

Recursive example

Iterative example

Parallel:

CUDA example (not optimized)

Bitonic sort features

- Data independent, no worst case
- Fast: $O(n \cdot \log^2 n)$ (Why?)
- Good locality in some parts

but

- Big leaps in addressing for some parts

What about those big leaps?

Small leaps: Can be computed within one block.
Shared memory friendly.

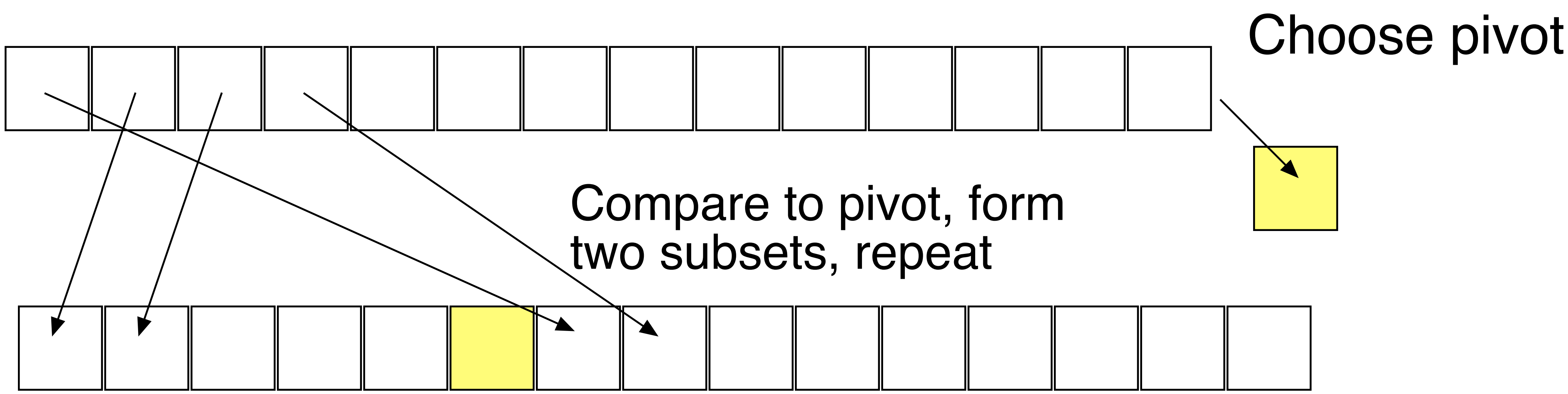
Big leaps (>number of threads/block): No synchronization possible between blocks!

But we must synchronize!

-> multiple kernel runs!

QuickSort

Very popular algorithm for sequential implementation



Data driven, data dependent reorganization, non-uniform

Fancy name - nobody expect QuickSort to be nothing but optimal

QuickSort is

Fast: O(n·logn) in typical cases

$O(n^2)$ in the worst case

Data driven, data dependent reorganization, non-uniform

QuickSort on GPU

Initially ignored as impractical

CUDA implementations exist

Data driven approaches increasingly suitable as GPUs become more flexible

Parallel QuickSort

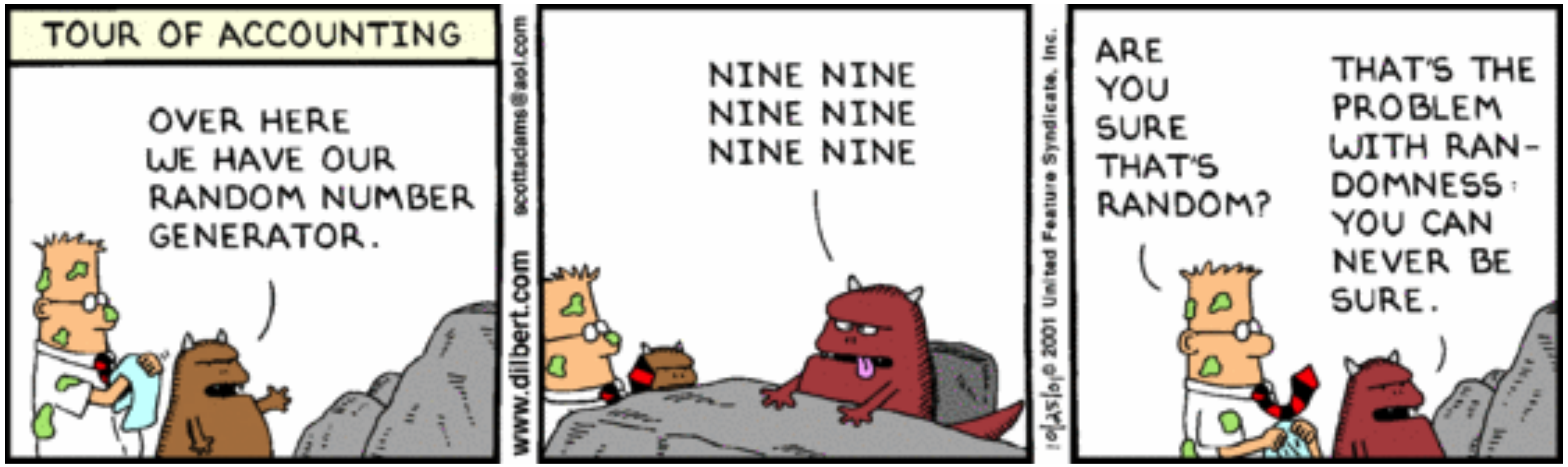
Several stages to consider:

- Pivot selection. Usually just grab one.
- Comparisons
- Partitioning
- Concatenate result

Pivot selection

If we could always pick a pivot that splits the data in half…

but you can't do that without sorting! (Or a histogram.) But how about a random one?



There is a worst case caused by bad pivots. Live with it!

Comparisons

Easy to parallelize

One thread per comparison not unreasonable! (GPUs don’t have a problem with many threads!)

No problem!

Partitioning

The big problem!

Sequential partitioning: Bad!

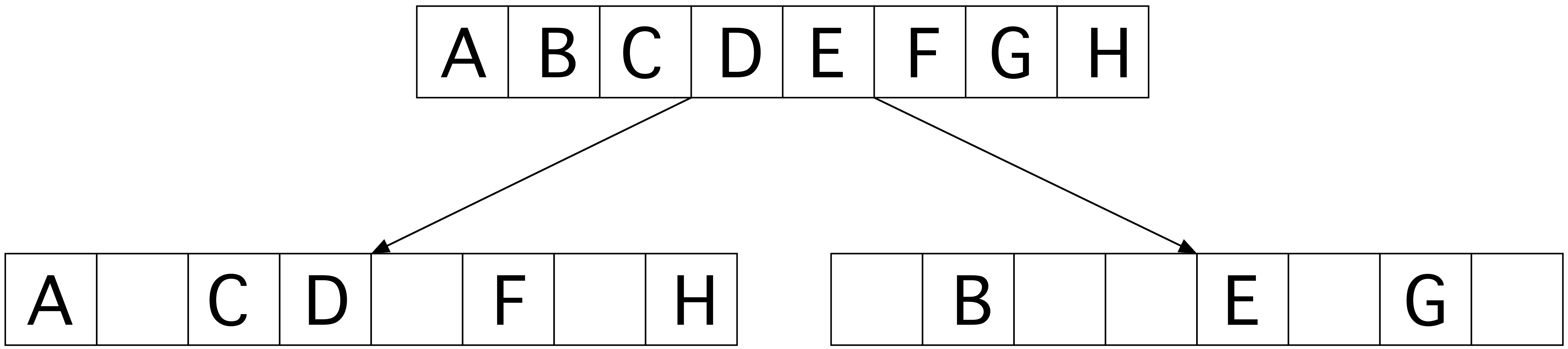
Parallel partitioning 1: Atomic fetch & increment.
(GPUs have atomics!)

Parallel partitioning 2: Divide and conquer

In-place sorting not feasible

Split to two list of same size as original. Massive number of threads!

Then we must pack to smaller size.



Packing to smaller size not trivial

Data dependent

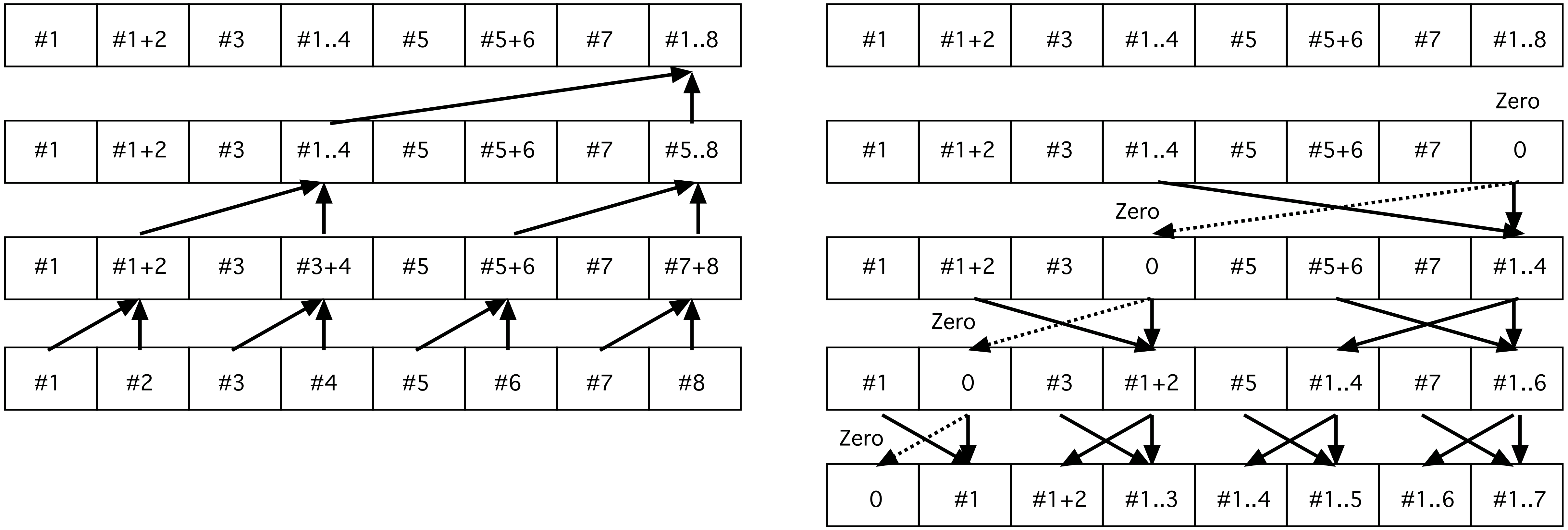
Use *parallel prefix sum* to create a look-up table for addressing.

Computes sum of all previous items.

Takes logN steps to perform.

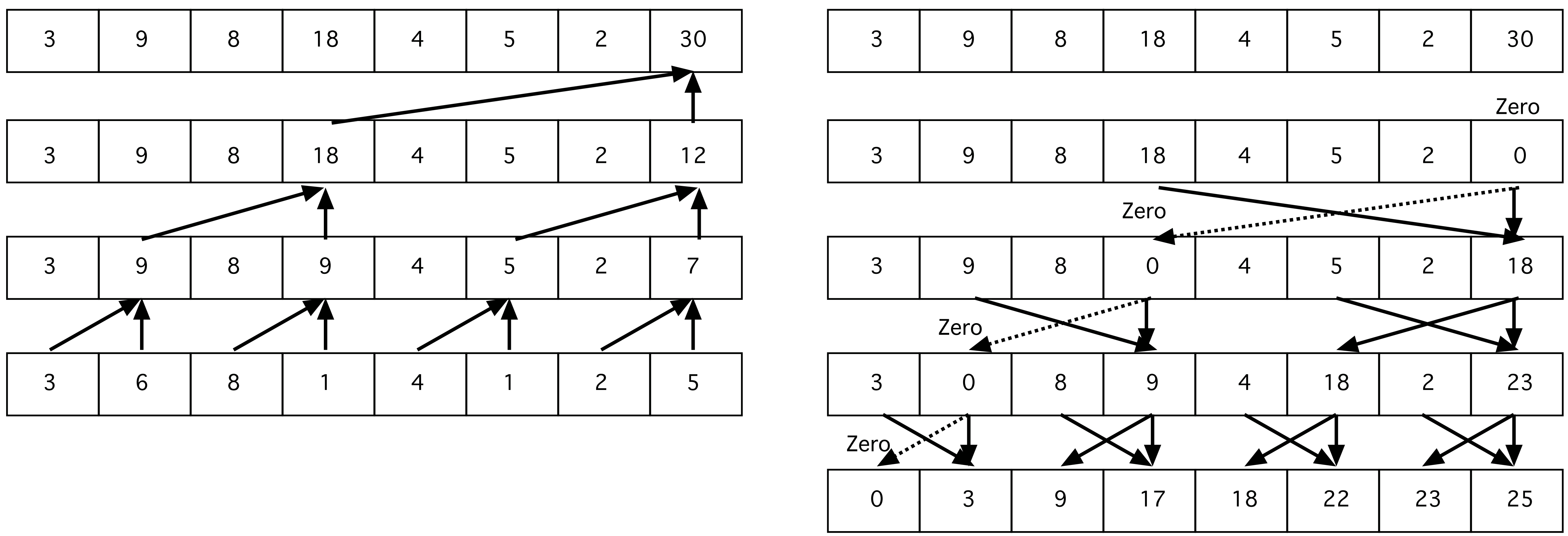
Parallel prefix sum

Similar to reduction but full output.

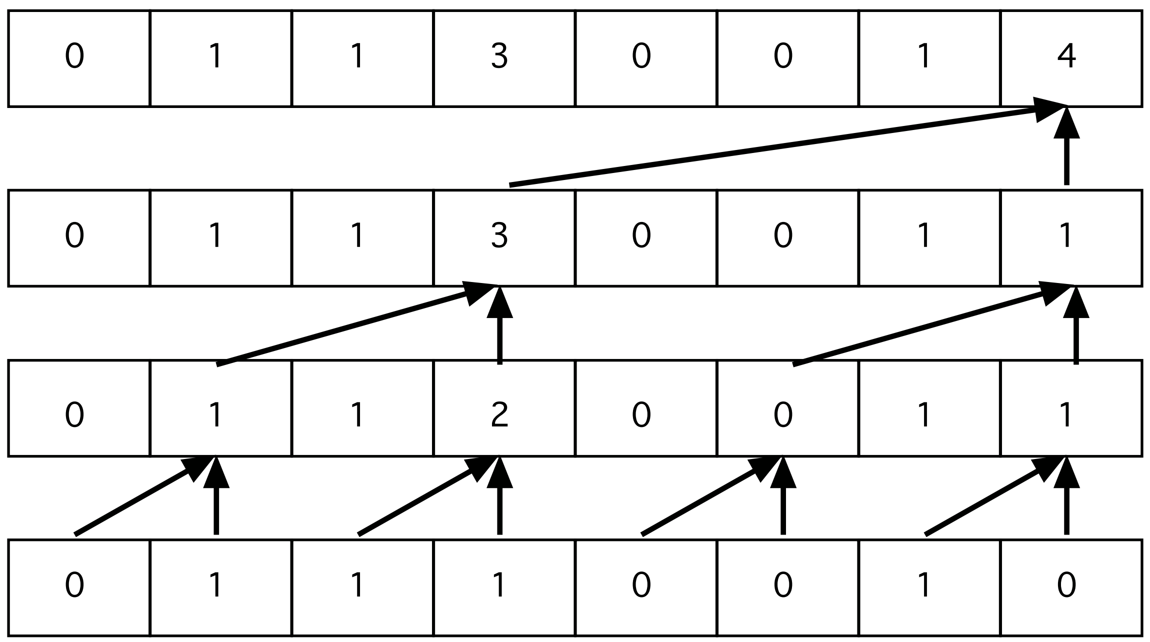


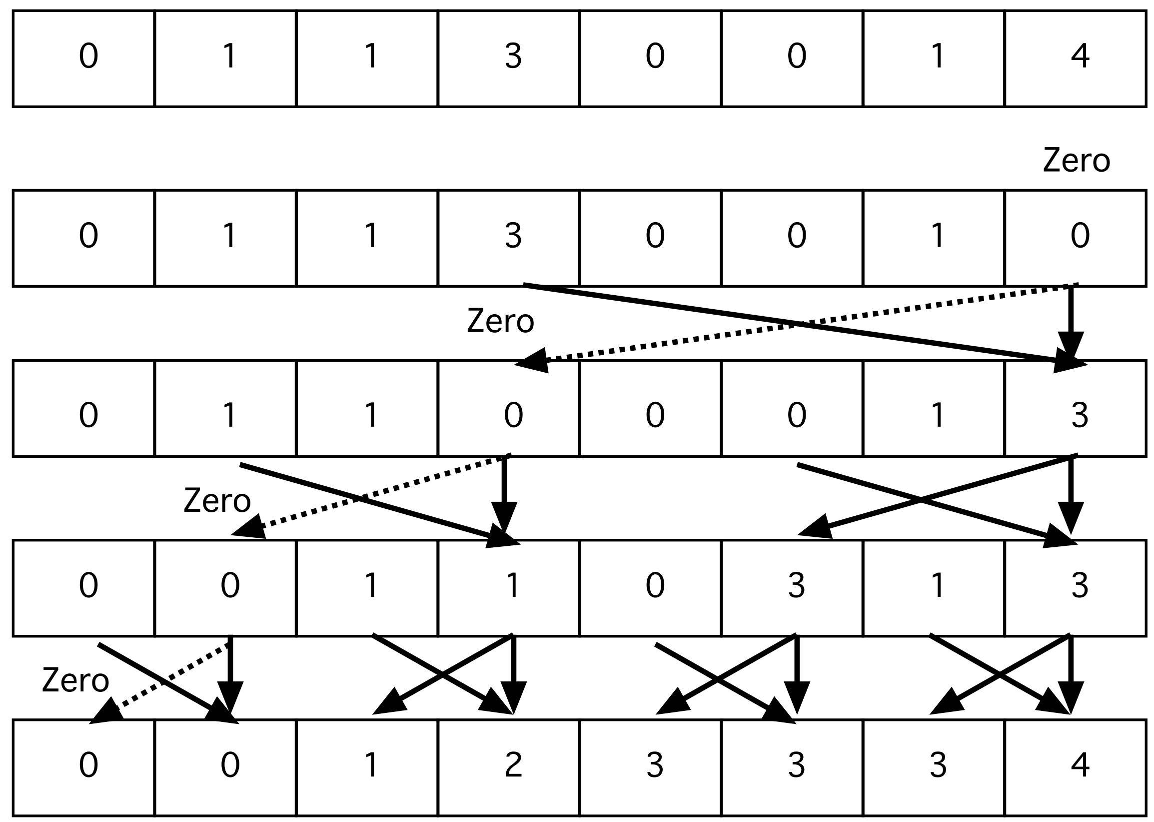
Parallel prefix sum

Example



For sorting: Binary parallel prefix sum





Parallel prefix sum on GPU

- No reason to use few threads. Use as many as you have output items.
- Multiple kernel runs to adapt to problem size variation.
- As described above, non-coalesced. Pack intermediate values for coalescing. If using shared memory, risk of bank conflicts. [Capannini]

Thus, QuickSort is not impossible, but more complex than before.

Note:

GPUs have Compare-And-Swap atomics!

GPUs favor massive numbers of threads. One thread per comparison is more than OK!

Implementations available. Example:

https://sourceforge.net/projects/cuda-quicksort/

Recursion

GPUs can’t do recursion efficiently… or can they?

Since Kepler we have concurrent kernels

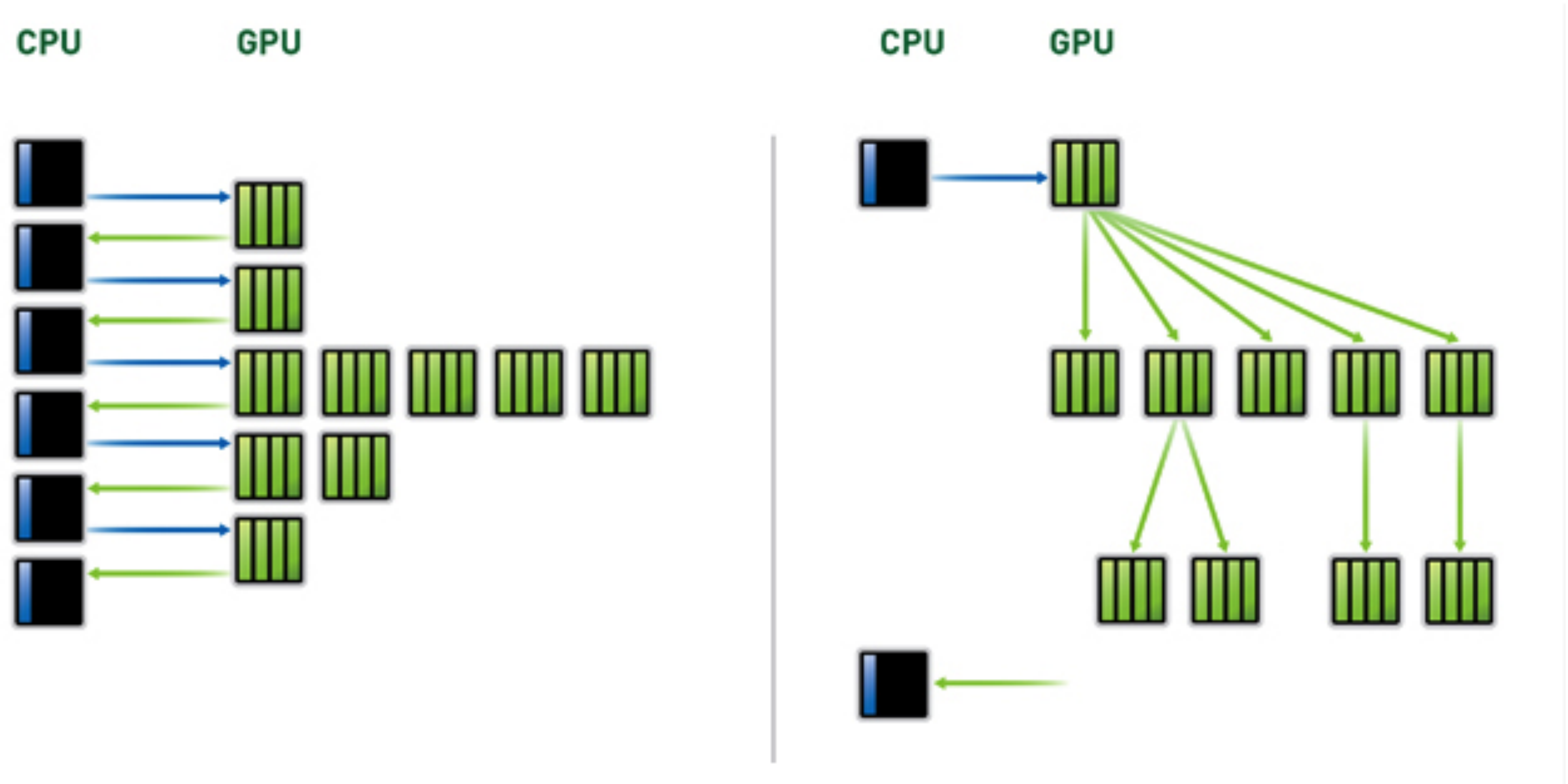
Not only a matter of launching kernels from CPU!

A kernel can spawn new kernels!

Do recursion by spawning new kernels!

Concurrent kernels, Dynamic Parallelism

Less work for the CPU to manage the computation.



Recursion can look like this:

```
__global__ void quicksort(int *data, int left, int right)
{
    int nleft, nright;
    cudaStream_t s1, s2;

    // Partitions data based on pivot of first element.
    // Returns counts in nleft & nright
    partition(data+left, data+right, data[left], nleft, nright);

    // If a sub-array needs sorting, launch a new grid for it.
    // Note use of streams to get concurrency between sub-sorts
    if(left < nright) {
        cudaStreamCreateWithFlags(&s1, cudaStreamNonBlocking);
        quicksort<<< ..., s1 >>>(data, left, nright);
    }
    if(nleft < right) {
        cudaStreamCreateWithFlags(&s2, cudaStreamNonBlocking);
        quicksort<<< ..., s2 >>>(data, nleft, right);
    }
}

__host__ void launch_quicksort(int *data, int count)
{
    quicksort<<< ... >>>(data, 0, count-1);
}
```

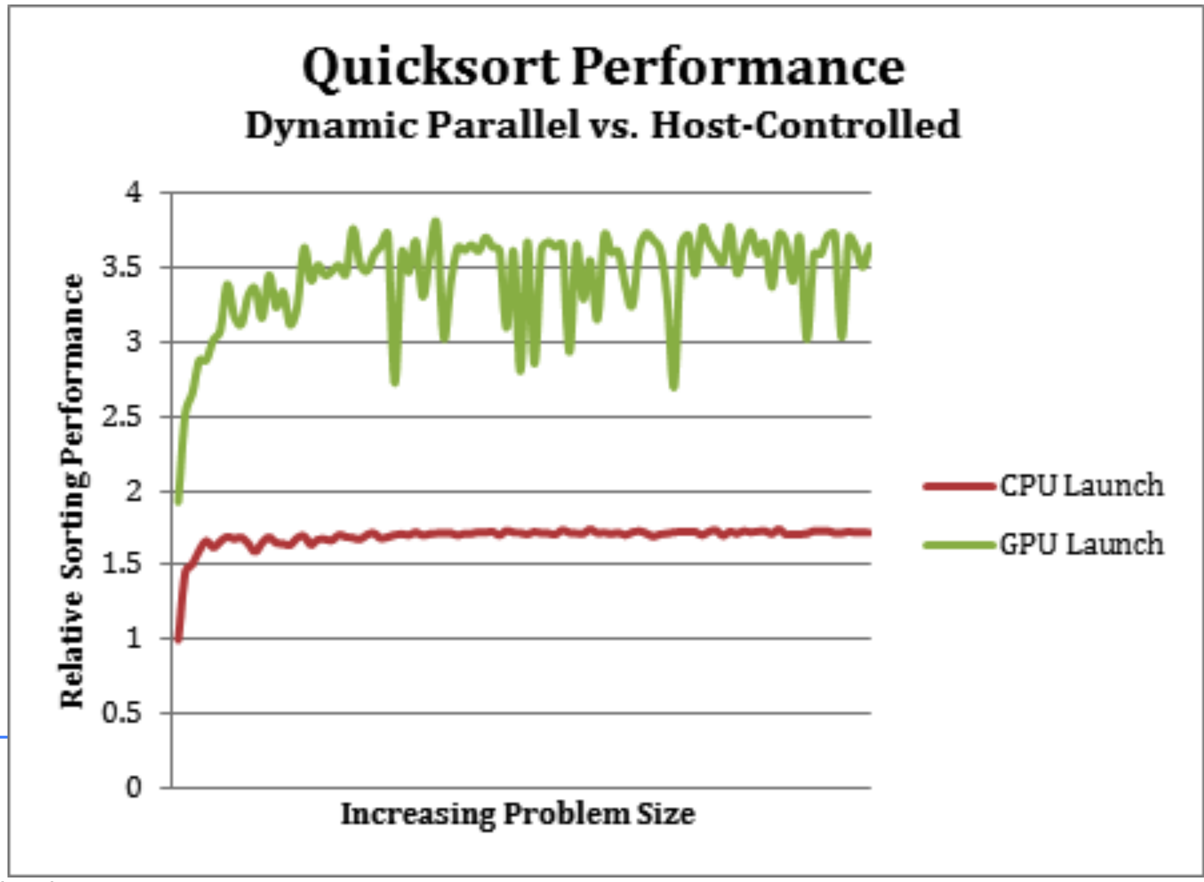
But… does this really do a good job on partitioning?

Source: http://blogs.nvidia.com/blog/2012/09/12/how-tesla-k20-speeds-up-quicksort-a-familiar-comp-sci-code/

Advantages

- Less work for CPU
- Less synchronizing (from CPU side)
- Easier programming!

They claim it matters this much (but your milage will vary)



Recursive CUDA kernels, a significant improvement, powerful option

Many other sorting algorithms exist... like this one this year:

Available online at www.sciencedirect.com

ScienceDirect

Procedia Computer Science 218 (2023) 1682–1691

Procedia Computer Science

www.elsevier.com/locate/procedia

International Conference on Machine Learning and Data Engineering

New GPU Sorting Algorithm Using Sorted Matrix

Sumit Kumar Gupta[a,*], Dr. Dhirendra Pratap Singh[a], Dr. Jaytrilok Choudhary[a]

[a] *Department of Computer Science and Engineering, Maulana Azad National Institute of Technology, Bhopal, India*

Other non-trivial algorithms

FFT, Fast Fourier Transform

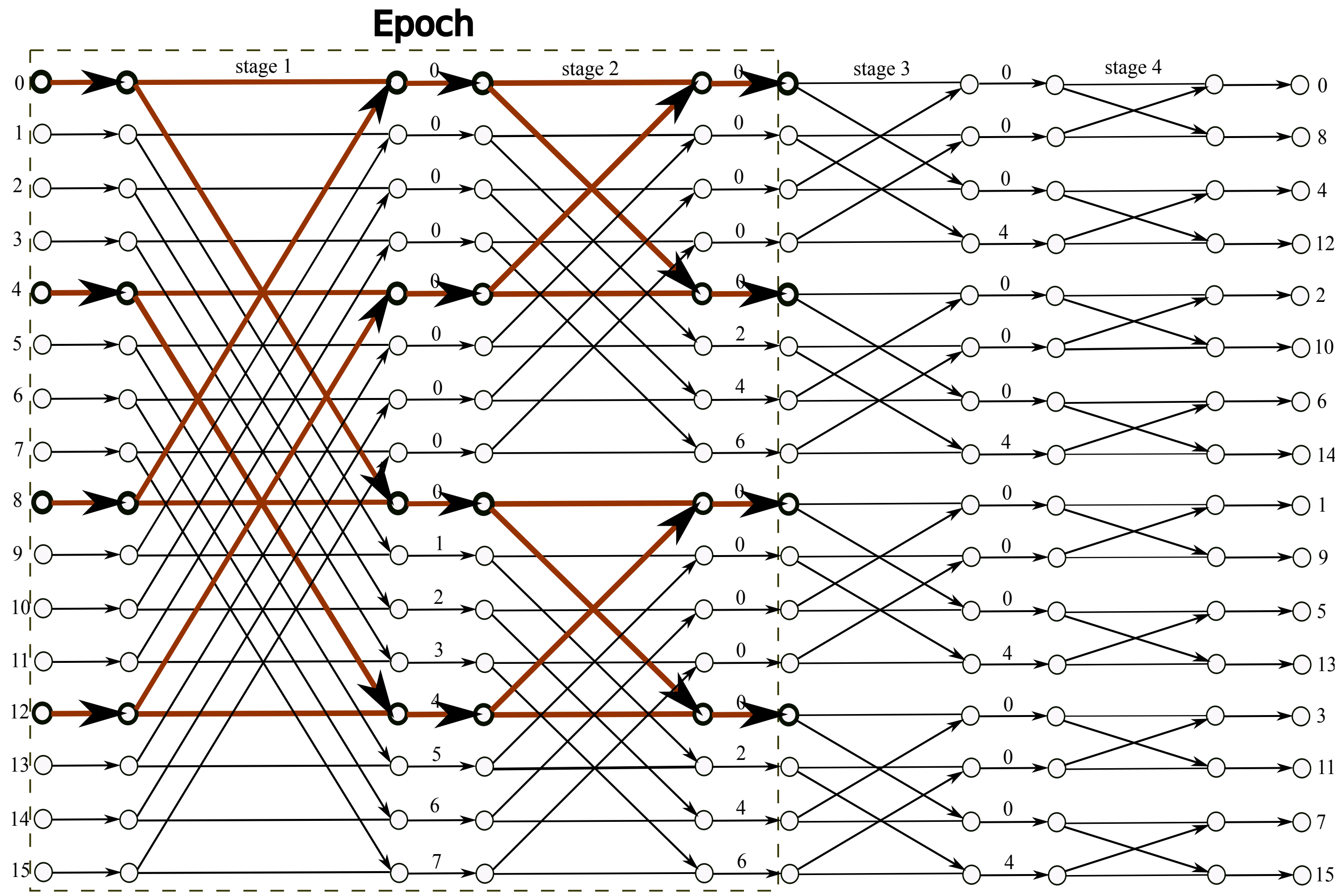
Distance transform

Fractal Brownian Motion

Fast Fourier Transform

Based on a sequence of "butterflies"

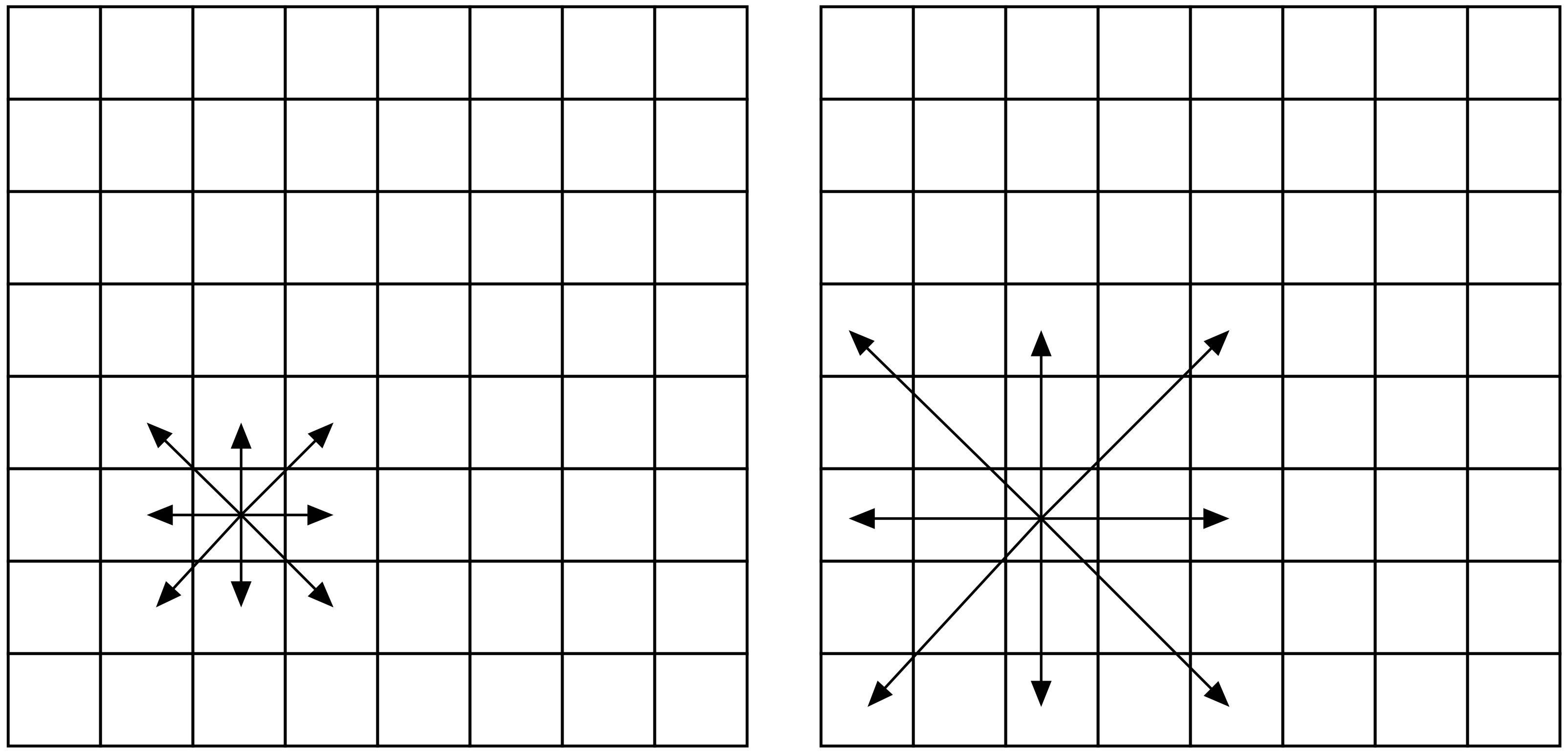
Similarily to Bitonic sort, can be computed several stage in one run for the "smaller" stages



Distance transform

Fast and simple version by Danielsson 1980: "Jump flooding"

Makes "jumps" of various length



Every "jump" needs to be one kernel run!

Fractal Brownian Motion

Used for e.g. realistic looking procedural terrains

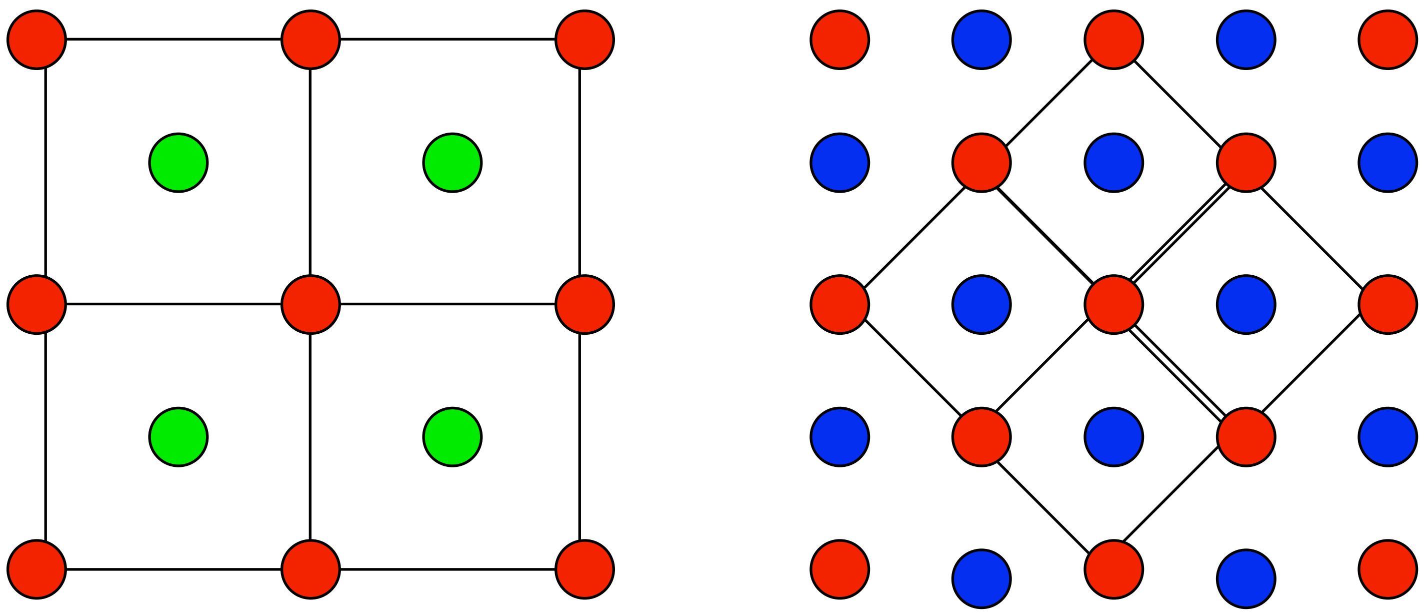
Among other methods:

- Diamond-square
- Multi-pass Perlin noise

Diamond-square algorithm

1) Midpoint from corners

2) Edge from corners and midpoints



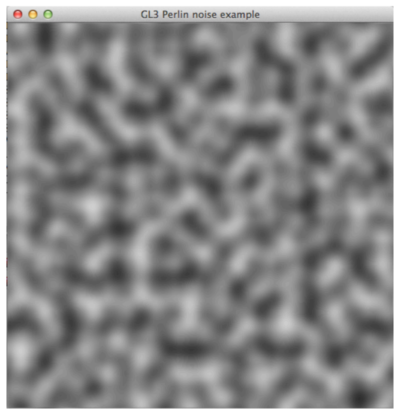
Repeat to desired resolution

Multi-pass Perlin noise

Theoretically slower than Diamond-square

BUT

can be computed by independent threads! One kernel run!



Single octave

FBM needs log N passes of different frequency

Conclusion

Algorithms with dependency in computed data often need multiple kernel runs.

This is an extra cost!

Does it pay when the computational complexity is lower?

That's all folks!